

First Midterm

Calculators are not allowed. Please turn off your mobile and pager.

1. (2 points) Find $\frac{dy}{dx}$ if $y = \frac{(\tanh x)^{\tan x} \cdot 3^{\ln(\sin^{-1} x)}}{e^{\sec(x^3)} \sqrt{2-x^3}}$

2. (5 points)

(a) Show that $\cos^{-1}\left(\frac{x-1}{x+1}\right) = 2 \cot^{-1}(\sqrt{x}) - \frac{\pi}{2}$.

(b) Solve, for x , the equation $\coth(\ln x) = 3$.

3. (3 points each) Evaluate the following integrals

(a) $\int \frac{e^x}{3 + e^{x+2}} dx$

(b) $\int_1^e \frac{1}{x(1 + \ln(\sqrt{x}))^{\frac{1}{3}}} dx$

(c) $\int \frac{\sin x}{\sqrt{1 + \sin^2 x}} dx$

(d) $\int \frac{5}{\tanh x \sqrt{\cosh^2 x - 5}} dx$

4. (2 points each) Let $f(x) = \cos^{-1}(4^{-x} - 3) + \frac{\pi}{2}$.

(a) Find the domain and the range of f .

(b) Show that f has an inverse function.

(c) Find $f^{-1}(x)$.